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ANALYSIS OF CONTACT STRESSES AND SLIP IN GEARS POWER SHIFT USING MULTICRITERIAL OPTIMISATION

ANALIZA NAPRĘŻEŃ KONTAKTOWYCH I POŚLIZGÓW MIĘDZYZĘBNYCH W PRZEKŁADNIACH ZĘBATYCH POWER SHIFT Z WYKORZYSTANIEM OPTYMALIZACJI WIELOKRYTERIALNEJ

Keywords: multicriterion optimisation, gear meshing, contact stress, slippage

A b s t r a c t

The work analysed contact stresses and slippage occurring in specific toothed pairs of two power shift gearings with eight ratios. The analysed gearings have same ratios of identical values. Difference in contact stresses and slippage values within individual toothed pairs of both gearings (at same values of input torque and engine speed) results from internal configuration of the kinematic chains created by the toothed gears within individual gear ratios. The analysis included 5 characteristic contact points within the tooth engagement area. They were selected analytically depending on geometrical parameters of gears that constitute the toothed pair. At computer-assisted design works that employ multi-criterion optimisation it is possible to minimize slippage and take reasonable advantage of fatigue contact durability of the material that was used for producing the toothed gears.

S t r e s z c z e n i e

W pracy analizowano naprężenia kontaktowe i poślizgi międzyzębne występujące w poszczególnych parach zębatych dwóch przekładni typu power shift o ośmiu stopniach przełożenia. Analizowane przekładnie zębate posiadają jednakową liczbę przełożeń oraz jednakowe ich wartości. Różnica w wartościach naprężeń kontaktowych i poślizgów międzyzębnych w poszczególnych parach zębatych obydwu przekładni, przy tych samych wartościach wejściowego momentu obrotowego i prędkości obrotowej, wynika z konfiguracji wewnętrznej łańcuchów kinematycznych utworzonych przez koła zębate, na poszczególnych stopniach przełożenia. Do analizy wybrano 5 charakterystycznych punktów przyporu w strefie zazębienia, które określono analitycznie w zależności od parametrów geometrycznych kół tworzących parę zębatą. W pracach projektowych wspomaganych komputerowo przy stosowaniu optymalizacji wielokryterialnej istnieje możliwość racjonalnego wykorzystania zmęczeniowej wytrzymałości kontaktowej materiału, z którego wykonane są koła zębate oraz minimalizacja poślizgów międzyzębnych.

Słowa kluczowe: optymalizacja wielokryterialna, zazębienie kół zębatych, naprężenia kontaktowe, poślizg międzyzębny

1. Introduction

Power shift gearings [1,2,3,4] are used in transmissions of engineering machines as they allow changing gear ratio at full load. Such a functionality is provided by toothed gears which remain in constant engagement as well as by multidisc clutches integrated with appropriate toothed gears. When a gear is changed, the load exerted on a toothed gear and clutch disks increases and slippage in the first stages of friction coupling within the clutch disks is inevitable.

Researches over contact stresses and slippage that employ multi-criterion optimisation and cover each toothed pair in a gearing make is possible to choose the geometrical parameters of the toothed gears which allow for using the fatigue contact durability σ_{Hlim} of the material that served for producing the toothed gears. The multi-criterion optimisation strives for minimizing contact stress and slippage at specific geometrical parameters of toothed gears.

2. Characteristics of the research subject

Researches were done on two power shift gearings [1] marked as PZA and PZB. Each of those gearings has eight gear ratios accomplished through specific wet multidisc clutches integrated with toothed gears. Kinematic schemes in axial alignment for PZA and PZB gearings were illustrated on figure 1.

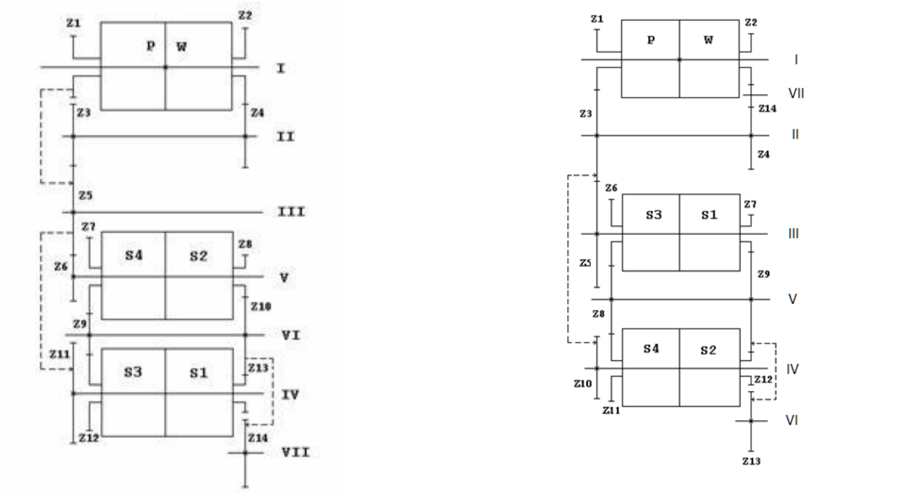

Fig. 1. Kinematic scheme in axial alignment of PZA gearing and PZB gearing
Rys. 1. Schemat kinematyczny w przekroju osiowym przekładni zębatej PZA i PZB

The PZA and PZB gearings feature 14 toothed gears located on 7 shafts. By engaging, the gears create 10 toothed pairs (kinematic pairs) which are positioned within appropriate kinematic chain from input shaft I to output shaft VII and denote gear ratios. Shafts I, IV and V (III for PZB gearing) feature the following clutches: P, W, S_1, S_2, S_3, S_4. The clutch P allows forward motion, while the clutch W allows backward motion. Clutches S_1, S_2, S_3, S_4 serve for maintaining ratios at stages 1-8.

The positioning of tooth gears inside a gearing affects tooth engagement between the given toothed gear and the nearest gears. A good indication of the tooth engagement cycles for any toothed gear is a kinematic scheme of the gearing in radial alignment, which has been presented in figure 2.

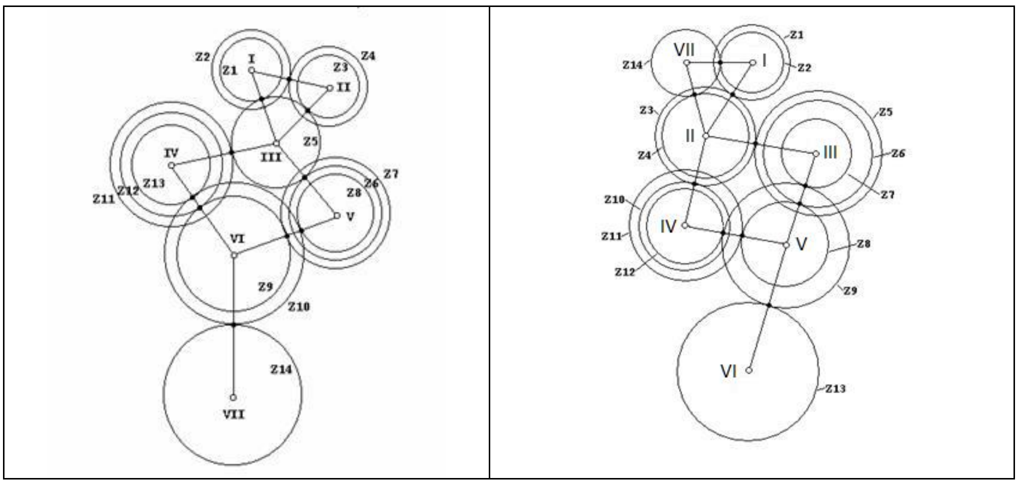

Fig. 2. Kinematic scheme in radial alignment of PZA gearing and PZB gearing

Basing on figures 1 and 2 it is possible to record the routes of kinematic chain within individual ratios in PZA and PZB gearings.

PZA gearing:

$$i_1 = \frac{z_5}{z_1} \bullet \frac{z_{11}}{z_5} \bullet \frac{z_{10}}{z_{13}} \bullet \frac{z_{14}}{z_{10}} \qquad i_5 = \frac{z_4}{z_2} \bullet \frac{z_5}{z_3} \bullet \frac{z_{11}}{z_5} \bullet \frac{z_{10}}{z_{13}} \bullet \frac{z_{14}}{z_{10}}$$

$$i_2 = \frac{z_5}{z_1} \bullet \frac{z_6}{z_5} \bullet \frac{z_{10}}{z_8} \bullet \frac{z_{14}}{z_{10}} \qquad i_6 = \frac{z_4}{z_2} \bullet \frac{z_5}{z_3} \bullet \frac{z_6}{z_5} \bullet \frac{z_{10}}{z_8} \bullet \frac{z_{14}}{z_{10}}$$

$$i_3 = \frac{z_5}{z_1} \bullet \frac{z_{11}}{z_5} \bullet \frac{z_9}{z_{12}} \bullet \frac{z_{14}}{z_{10}} \qquad i_7 = \frac{z_4}{z_2} \bullet \frac{z_5}{z_3} \bullet \frac{z_{11}}{z_5} \bullet \frac{z_9}{z_{12}} \bullet \frac{z_{14}}{z_{10}}$$

$$i_4 = \frac{z_5}{z_1} \bullet \frac{z_6}{z_5} \bullet \frac{z_9}{z_7} \bullet \frac{z_{14}}{z_{10}} \qquad i_8 = \frac{z_4}{z_2} \bullet \frac{z_5}{z_3} \bullet \frac{z_6}{z_5} \bullet \frac{z_9}{z_7} \bullet \frac{z_{14}}{z_{10}}$$

PZB gearing:

$$i_1 = \frac{z_3}{z_1} \bullet \frac{z_5}{z_3} \bullet \frac{z_9}{z_7} \bullet \frac{z_{13}}{z_9} \qquad i_5 = \frac{z_{14}}{z_2} \bullet \frac{z_4}{z_{14}} \bullet \frac{z_5}{z_3} \bullet \frac{z_9}{z_7} \bullet \frac{z_{13}}{z_9}$$

$$i_2 = \frac{z_3}{z_1} \bullet \frac{z_{10}}{z_3} \bullet \frac{z_9}{z_{12}} \bullet \frac{z_{13}}{z_9} \qquad i_6 = \frac{z_{14}}{z_2} \bullet \frac{z_4}{z_{14}} \bullet \frac{z_{10}}{z_3} \bullet \frac{z_9}{z_{12}} \bullet \frac{z_{13}}{z_9}$$

$$i_3 = \frac{z_3}{z_1} \bullet \frac{z_5}{z_3} \bullet \frac{z_8}{z_6} \bullet \frac{z_{13}}{z_9} \qquad i_7 = \frac{z_{14}}{z_2} \bullet \frac{z_4}{z_{14}} \bullet \frac{z_5}{z_3} \bullet \frac{z_8}{z_6} \bullet \frac{z_{13}}{z_9}$$

$$i_4 = \frac{z_3}{z_1} \bullet \frac{z_{10}}{z_3} \bullet \frac{z_8}{z_{11}} \bullet \frac{z_{13}}{z_9} \qquad i_8 = \frac{z_{14}}{z_2} \bullet \frac{z_4}{z_{14}} \bullet \frac{z_{10}}{z_3} \bullet \frac{z_8}{z_{11}} \bullet \frac{z_{13}}{z_9}$$

Ratios i_1 to i_4 allow forward motion of the machine, while ratios i_5 to i_8 provide the backward motion. The gear which is characterized by the most tooth engagement cycles has been selected from within the internal structure of the considered gearings. In case of the PZA gearing, it is the z_5 gear of tooth engagement cycles equalling to 4, while the z_3 gear in the PZB gearing has 3 tooth engagement cycles. The teeth engagement cycles were illustrated in figure 3.

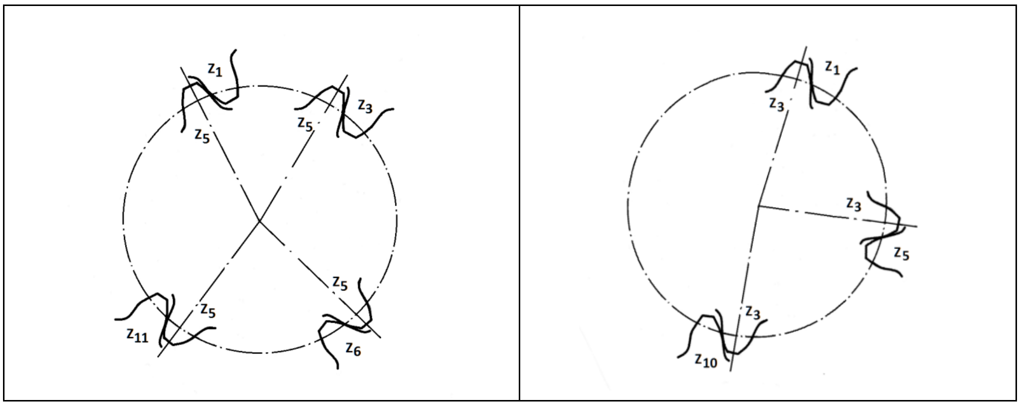

Fig. 3. Teeth engagement cycles in individual gearings: PZA gearing and PZB gearing

In case of the PZA gearing, the tooth engagement cycles of z_5 gear are indicated by toothed gears z_1, z_3, z_6, $z_{11,}$ while tooth engagement cycles of z_3 gear are indicated by toothed gears z_1, z_5, z_{10}. During the time of operation assumed for the PZA gearing, the highest number of load cycles will be accomplished by the gear z_5. This indicates that pitting will probably be the initial cause of fatigue wear. In the PZB gearing, toothed gear z_3 suffers the highest number of load cycles at the given time. Hence, this gear will be the first to suffer undesired effects of fatigue wear.

3. Numerical tests on contact stresses and slippage

The contact stresses and slippage have been calculated within the characteristic contact points at the active surface of tooth engagement. Characteristic contact points located on the sides of teeth [5,6,7,8,10] were presented on figure 4.

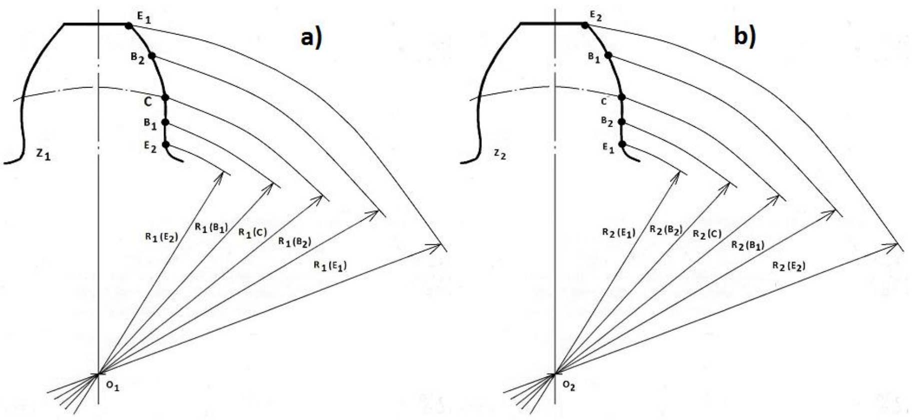

Fig. 4. Characteristic contact points: a) on the tooth of the driver gear; b) on the tooth of the follower gear

During tooth engagement, the characteristic points of teeth in the driver (active) gear and the follower (passive) gear overlap each other, which can be recorded as follows: $E_{2cz} = E_{2bier}$, $B_{1cz} = B_{1bier}$, $C_{cz} = C_{bier}$, $B_{2cz} = B_{2bier}$, $E_{1cz} = E_{1bier}$. Considering engagement of the driver (active) gear and the follower (passive) gear it can be stated that part of the involute profile of tooth E_2B_1 at the tooth root as well as part of the involute profile of tooth E_2B_1 at the tooth tip are forming an area of double-tooth engagement. Part of the involute profile B_2E_1 is also located within the double-tooth engagement, at the tooth tip of the driver (active) gear. It is positioned together with the involute profile B_2E_1, at the tooth root of the follower (passive) gear.

Contact stresses and slippage have been measured with an original computer software [9] at each of the five characteristic contact points, at an input load moment of 1500 [Nm] and input engine speed of 1600 [min^{-1}]. Due to their multiplicity, however, tables 1-4 present stress values only for the extreme points E_1 and E_2 obtained after the first calculation stage (before optimisation) and after the optimisation, for PZA and PZB gearings respectively. Table 1 presents contact stress values after the first stage of calculations within E_1 and E_2 points in the PZA gearing.

Table 1

Contact stress values [MPa] for E_1 and E_2 points in the PZA gearing before optimisation

ratio	toothed pair										contact point
	z_5/z_1	z_{11}/z_5	z_{10}/z_{13}	z_{14}/z_{10}	z_6/z_5	z_{10}/z_8	z_9/z_{12}	z_9/z_7	z_4/z_2	z_5/z_3	
i_1	632.9	849.6	934.5	1161.2							
i_2	632.9			1038.8	1037.3	586.2					
i_3	632.9	849.6		899.5			948.5				
i_4	632.9			734.9	1037.3			673.6			E_1
i_5		850.0	935.0	1161.9					681.1	617.3	
i_6				1039.3	1037.9	586.6			681.1	617.3	
i_7		850.0		900.0			949.0		681.1	617.3	
i_8				735.3	1037.9			674.0	681.1	617.3	
i_1	681.0	914.2	1005.5	1249.5							
i_2	681.0			1117.7	1116.1	630.8					
i_3	681.0	914.2		967.8			1020.6				
i_4	681.0			790.7	1116.1			724.8			
i_5		914.6	1006.1	1250.1					732.9	664.2	E_2
i_6				1118.3	1116.7	631.1			732.9	664.2	
i_7		914.6		968.4			1021.1		732.9	664.2	
i_8				791.1	1116.7			725.2	732.9	664.2	

Table 2 presents results from the first calculation stage of PZB gearing at the same load M=1500 [Nm], n = 1600 [min^{-1}].

Table 2

Contact stress values [MPa] for E_1 and E_2 points in the PZB gearing before optimisation

ratio	toothed pair										contact point
	z_3/z_1	z_5/z_3	z_9/z_7	z_{13}/z_9	z_{10}/z_3	z_9/z_{12}	z_8/z_6	z_8/z_{11}	z_{14}/z_2	z_4/z_{14}	
i_1	712.7	556.7	956.0	1112.3							
i_2	712.7			994.9	603.9	726.8					
i_3	712.7	556.7		824.9			1175.2				E_1
i_4	712.7			710.0	603.9			984.1			
i_5		556.7	956.0	1112.3					702.7	648.0	

i_6				994.9	603.9	726.8			702.7	648.0	
i_7		556.7		824.9			1175.2		702.7	648.0	
i_8				710.0	603.9			984.1	702.7	648.0	
i_1	777.1	607.0	1042.3	1212.7							E_2
i_2	777.1			1084.7	658.4	792.5					
i_3	777.1	607.0		899.4			1281.3				
i_4	777.1			774.1	658.4			1073.0			
i_5		607.0	1042.3	1212.7					766.2	706.5	
i_6				1084.7	658.4	792.5			766.2	706.5	
i_7		607.0		899.4			1281.3		766.2	706.5	
i_8				774.1	658.4			1073.0	766.2	706.5	

Results of contact stresses and slippage [9] have been obtained during multi-criterion optimisation with 11 criteria that include: maximum number of contact points, minimal tooth shape coefficient, minimal thickness at the tooth tip, total weight of toothed gears, total mass inertial moment of toothed gears, maximal durability of tooth root and tooth edge, material effort uniformity within toothed gears, minimal relative thickness of the oil film within the area between teeth, gearing efficiency and minimal slippage value.

Table 3 presents contact stresses values which were obtained during the PZA gearing optimisation that included 11 criteria.

Table 3

Contact stress values [MPa] for E_1 and E_2 points in the PZA gearing after optimisation

ratio	toothed pair										contact point
	z_5/z_1	z_{11}/z_5	z_{10}/z_{13}	z_{14}/z_{10}	z_6/z_5	z_{10}/z_8	z_9/z_{12}	z_9/z_7	z_4/z_2	z_5/z_3	
i_1	892.7	1273.2	1032.5	1271.2							E_1
i_2	892.7			1137.1	1410.3	669.3					
i_3	892.7	1273.2		984.7			1376.1				
i_4	892.7			804.5	1410.3			927.8			
i_5		1273.9	1033.1	1271.9					1192.5	846.8	
i_6				1137.8	1411.1	669.7			1192.5	846.8	
i_7		1273.9		985.2			1376.9		1192.5	846.8	
i_8				804.9	1411.1			928.3	1192.5	846.8	
i_1	969.0	1382.0	1120.7	1379.8							E_2
i_2	969.0			1234.3	1530.8	726.5					
i_3	969.0	1382.0		1068.8			1493.7				
i_4	969.0			873.2	1530.8			1007.1			
i_5		1382.7	1121.3	1380.6					1294.4	919.2	
i_6				1235.0	1531.7	726.9			1294.4	919.2	
i_7		1382.7		1069.4			1494.5		1294.4	919.2	
i_8				873.7	1531.7			1007.6	1294.4	919.2	

Results of optimization calculations for the PZB gearing, which were obtained accordingly to the PZA gearing, are presented in table 4.

Table 4

Contact stress values [MPa] for E_1 and E_2 points in the PZB gearing after optimisation

ratio	toothed pair										contact point
	z_3/z_1	z_5/z_3	z_9/z_7	z_{13}/z_9	z_{10}/z_3	z_9/z_{12}	z_8/z_6	z_8/z_{11}	z_{14}/z_2	z_4/z_{14}	
i_1	866.9	695.3	1106.6	1294.5							E_1
i_2	866.9			1157.9	702.9	864.3					
i_3	866.9	695.3		960.1			1474.9				

i_4	**866.9**			**826.3**	**702.9**			**1197.4**			
i_5		**695.3**	**1106.6**	**1294.5**					**852.7**	**871.5**	
i_6				**1157.9**	**702.9**	**864.3**			**852.7**	**871.5**	
i_7		**695.3**		**960.1**			**1474.9**		**852.7**	**871.5**	
i_8				**826.3**	**702.9**			**1197.4**	**852.7**	**871.5**	
i_1	**936.5**	**751.2**	**1195.5**	**1398.6**							E_2
i_2	**936.5**			**1250.9**	**759.4**	**933.7**					
i_3	**936.5**	**751.2**		**1037.2**			**1593.4**				
i_4	**936.5**			**892.7**	**759.4**			**1293.6**			
i_5		**751.2**	**1195.5**	**1398.6**					**921.2**	**941.5**	
i_6				**1250.9**	**759.4**	**933.7**			**921.2**	**941.5**	
i_7		**751.2**		**1037.2**			**1593.4**		**921.2**	**941.5**	
i_8				**892.7**	**759.4**			**1293.6**	**921.2**	**941.5**	

The maximum slippage values occur in the extreme points E_1 and E_2. The numerical values for PZA gearing obtained in the first calculation stage are presented in table 5.

Table 5

Slippage values ($m \times s^{-1}$) for E_1 and E_2 points in the PZA gearing before optimisation

ratio	toothed pair										contact point
	z_5/z_1	z_{11}/z_5	z_{10}/z_{13}	z_{14}/z_{10}	z_6/z_5	z_{10}/z_8	z_9/z_{12}	z_9/z_7	z_4/z_2	z_5/z_3	
i_1	**2.639**	**2.737**	**1.117**	**1.099**							E_1
i_2	**2.639**			**1.374**	**2.727**	**3.257**					
i_3	**2.639**	**2.737**		**1.832**			**2.283**				
i_4	**2.639**			**2.746**	**2.727**			**2.160**			
i_5		**2.732**	**1.116**	**1.097**					**7.277**	**2.420**	
i_6				**1.372**	**2.722**	**3.252**			**7.277**	**2.420**	
i_7		**2.732**		**1.830**			**2.281**		**7.277**	**2.420**	
i_8				**2.740**	**2.722**			**2.156**	**7.277**	**2.420**	
i_1	**2.880**	**1.368**	**2.006**	**1.361**							E_2
i_2	**2.880**			**1.701**	**2.364**	**1.727**					
i_3	**2.880**	**1.368**		**2.269**			**2.575**				
i_4	**2.880**			**3.401**	**2.364**			**5.696**			
i_5		**1.365**	**2.003**	**1.359**					**2.918**	**2.681**	
i_6				**1.699**	**2.359**	**1.724**			**2.918**	**2.681**	
i_7		**1.365**		**2.266**			**2.572**		**2.918**	**2.681**	
i_8				**3.394**	**2.359**			**5.686**	**2.918**	**2.681**	

Table 6 presents slippage values after the first calculation cycle for the PZB gearing and its extreme points E_1 and E_2.

Table 6

Slippage values ($m \times s^{-1}$) for E_1 and E_2 points in the PZB gearing before optimisation

ratio	toothed pair										contact point
	z_3/z_1	z_5/z_3	z_9/z_7	z_{13}/z_9	z_{10}/z_3	z_9/z_{12}	z_8/z_6	z_8/z_{11}	z_{14}/z_2	z_4/z_{14}	
i_1	**3.293**	**3.319**	**1.942**	**1.212**							E_1
i_2	**3.293**			**1.514**	**3.837**	**3.223**					
i_3	**3.293**	**3.319**		**2.203**			**3.015**				
i_4	**3.293**			**2.976**	**3.837**			**2.851**			
i_5		**3.317**	**1.942**	**1.212**					**4.583**	**4.684**	
i_6				**1.514**	**3.835**	**3.221**			**4.583**	**4.684**	
i_7		**3.317**		**2.203**			**3.015**		**4.583**	**4.684**	
i_8				**2.974**	**3.835**			**2.849**	**4.583**	**4.684**	
i_1	**3.512**	**2.250**	**1.597**	**1.766**							E_2

i_2	**3.512**			**2.207**	**2.427**	**1.980**					
i_3	**3.512**	**2.250**		**3.210**			**2.603**				
i_4	**3.512**			**4.337**	**2.427**			**5.108**			
i_5		**2.249**	**1.597**	**1.766**					**3.890**	**4.248**	
i_6				**2.207**	**2.425**	**1.978**			**3.890**	**4.248**	
i_7		**2.249**		**3.210**			**2.603**		**3.890**	**4.248**	
i_8				**4.334**	**2.425**			**5.104**	**3.890**	**4.248**	

Apart from the contact stresses and due to 11 criteria mentioned above, optimisation calculations in the PZA gearing also included the slippage in points E_1 and E_2. The slippage values are presented in table 7.

T a b l e 7

Slippage values ($m\times s^{-1}$) for E_1 and E_2 points in the PZA gearing after optimisation

ratio	**toothed pair**										**contact point**
	z_5/z_1	z_{11}/z_5	z_{10}/z_{13}	z_{14}/z_{10}	z_6/z_5	z_{10}/z_8	z_9/z_{12}	z_9/z_7	z_4/z_2	z_5/z_3	
i_1	**2.871**	**2.554**	**1.315**	**1.143**							
i_2	**2.871**			**1.429**	**2.863**	**2.979**					
i_3	**2.871**	**2.554**		**1.905**			**2.433**				
i_4	**2.871**			**2.856**	**2.863**			**3.248**			**E_1**
i_5		**2.551**	**1.314**	**1.141**					**2.962**	**3.097**	
i_6				**1.427**	**2.860**	**2.976**			**2.962**	**3.097**	
i_7		**2.551**		**1.903**			**2.430**		**2.962**	**3.097**	
i_8				**2.852**	**2.860**			**3.244**	**2.962**	**3.097**	
i_1	**3.079**	**1.642**	**2.085**	**1.518**							
i_2	**3.079**			**1.898**	**3.013**	**2.134**					
i_3	**3.079**	**1.642**		**2.530**			**1.655**				
i_4	**3.079**			**3.793**	**3.013**			**4.412**			
i_5		**1.641**	**2.082**	**1.515**					**3.989**	**2.936**	**E_2**
i_6				**1.895**	**3.010**	**2.132**			**3.989**	**2.936**	
i_7		**1.641**		**2.528**			**1.653**		**3.989**	**2.936**	
i_8				**3.788**	**3.010**			**4.407**	**3.989**	**2.936**	

The optimization calculations of slippage in contact points E_1 and E_2 in the PZB gearing were illustrated in table 8.

T a b l e 8

Slippage values ($m\times s^{-1}$) for E_1 and E_2 points in the PZB gearing after optimisation

ratio	**toothed pair**										**contact point**
	z_3/z_1	z_5/z_3	z_9/z_7	z_{13}/z_9	z_{10}/z_3	z_9/z_{12}	z_8/z_6	z_8/z_{11}	z_{14}/z_2	z_4/z_{14}	
i_1	**3.920**	**3.504**	**2.048**	**1.295**							
i_2	**3.920**			**1.618**	**3.758**	**2.987**					
i_3	**3.920**	**3.504**		**2.354**			**2.850**				
i_4	**3.920**			**3.180**	**3.758**			**2.991**			**E_1**
i_5		**3.504**	**2.048**	**1.295**					**3.537**	**3.908**	
i_6				**1.618**	**3.758**	**2.987**			**3.537**	**3.908**	
i_7		**3.504**		**2.354**			**2.850**		**3.537**	**3.908**	
i_8				**3.180**	**3.758**			**2.991**	**3.537**	**3.908**	
i_1	**3.796**	**3.093**	**1.544**	**1.744**							
i_2	**3.796**			**2.180**	**3.644**	**2.073**					
i_3	**3.796**	**3.093**		**3.171**			**2.251**				**E_2**
i_4	**3.796**			**4.284**	**3.644**			**4.765**			
i_5		**3.093**	**1.544**	**1.744**					**3.791**	**3.848**	
i_6				**2.180**	**3.644**	**2.073**			**3.791**	**3.848**	

i_7		**3.093**		**3.171**			**2.251**		**3.791**	**3.848**	
i_8				**4.284**	**3.644**			**4.765**	**3.791**	**3.848**	

Optimization of the PZA and PZB gearings has been conducted in relation to 11 criteria referenced by the global criterion. This work focuses on contact stresses and slippage. It is believed that multi-criterion optimisation is an optimal solution when global criterion reaches the minimum. Figure 5 presents the contact stress chart in relation to optimisation steps for the PZA gearing.

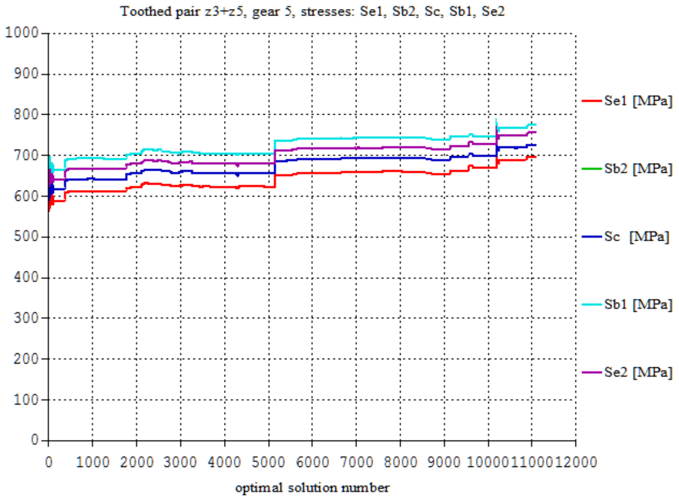

Fig. 5. Contact stresses in 5 characteristic points for the PZA gearing

Figure 6 presents the contact stress chart in relation to optimisation steps for the PZB gearing.

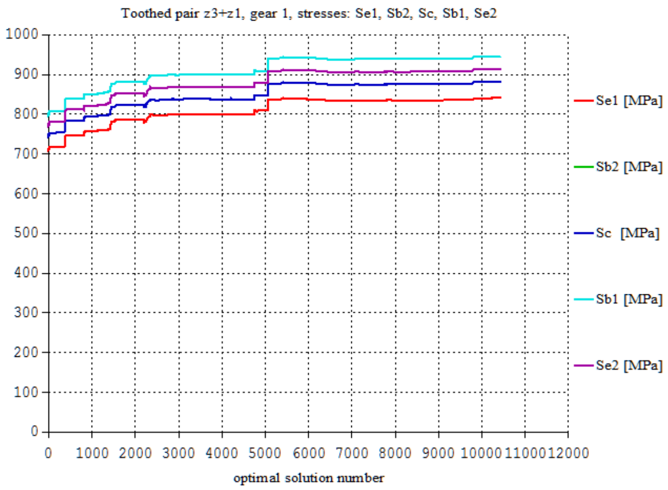

Fig. 6. Contact stresses in 5 characteristic points for the PZB gearing

4. Analysis of the results

For each ratio from i_1 to i_8 for contact points E_1 and E_2 , contact stresses in toothed pair z_{14}/z_{10} in the PZA gearing presented in table 1 are higher than contact stresses in toothed pair z_{13}/z_9 in the PZB gearing illustrated in table 2. The purpose for comparing stress values between toothed pairs z_{14}/z_{10} and z_{13}/z_9 is justified by the fact that they constitute the last pair in the kinematic chain of ratios from i_1 to i_8.

According to tables 1 and 3, comparison of contact stresses within toothed pairs z_5/z_1, z_{11}/z_5, z_6/z_5, z_5/z_3 with 4 engagement cycles (being a part of the gear z_5 in the PZA gearing) between contact stresses within toothed pairs z_3/z_1, z_5/z_3, z_{10}/z_3 with 3 engagement cycles (being a part of the gear z_3 in the PZB gearing) indicates that contact stresses in toothed gear z_5 in the PZA gearing are higher. Higher stress values and four tooth engagement cycles in toothed gear z_5 indicate that after a given time of operation this gear will suffer the highest number of load cycles. From this reason, it can be expected that first occurrences of fatigue contact durability take the form of pitting.

According to table 5, after a single calculation stage the slippage in toothed pairs z_5/z_1, z_{11}/z_5, z_6/z_5, z_5/z_3 in the PZA gearing does not significantly differ from slippage in the same toothed pairs following optimisation according to table 7. According to tables 5 and 7, the highest difference between slippage value after a single calculation stage and the value obtained after optimisation has been displayed by toothed pair z_4/z_2 in the PZA gearing at ratios from i_5 to i_8. Following optimisation, slippage in this toothed pair has decreased from 7.277 $m \times s^{-1}$ to 2.962 $m \times s^{-1}$.

The PZB gearing is characterized by higher slippage values within all ratios in each toothed pair. The toothed pairs z_{14}/z_2 and z_4/z_{14} in the PZB gearing described in table 6 display the highest slippage values after the first calculation stage within ratios i_5 to i_8 and point E_1 as well as within ratios i_5 to i_8 and point E_2. By comparing slippage values presented in tables 6 and 8 it can be noticed that toothed pairs: z_3/z_4, z_5/z_3, z_{10}/z_3 display increase in slippage after optimization calculations.

5. Summary

The analysis of calculations with multi-criterion optimisation conducted on the PZA and PZB gearings showed that toothed pairs of the both gearings do not display significant differences in slippage values after optimisation.

The criterion of contact stresses indicates that configuration of the PZB gearing is more favourable than configuration of the PZA gearing. The toothed gear z_3 being a part of its structure is characterized by 3 engagement cycles, while the toothed gear z_5 in the PZA gearing has four engagement cycles. Thus, the expected durability of the PZB gearing will be greater due to fatigue contact durability of the toothed gear edges.

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